A GENERALIZED METHOD FOR MAKING OPTIMAL ESTIMATES OF RESERVOIR-AQUIFER PARAMETERS

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The present study presents a method of predicting performance of reservoir-aquifer systems and for making optimal estimates of aquifer geometry, reservoir size, aquifer size, and reservoir-aquifer fluid conductivity.

Analytical solution for linear aquifer connected to a reservoir with constant compressibility is presented. A new method is derived to predict behaviour of a reservoir with changing compressibility using this analytical solution for a linear aquifer or the corresponding analytical solution for a (infinite) radial aquifer. The accuracy of this new method was verified by comparing calculated results with results calculated using a more exact finite difference simulator. The advantage of the new method is a considerable reduction in computer time required.

Once verified the new prediction method was incorporated into a computer program for estimating reservoir parameters.

The program uses a modified efficient optimization technique to determine parameters which result in best agreement between observed and calculated reservoir pressure. Changing from a linear to a radial aquifer requires simply changing a table in the program. The program was applied to an actual field. This application compared results obtained with linear and radial aquifers and indicated that in addition to aquifer geometry, reservoir size, aquifer size and reservoir-aquifer fluid conductivity are key parameters in predicting performance of a reservoir-aquifer system.

Results of application of the method to an actual oil field are given and comparison is made with results obtained using the Hurst-Van Everdingen water influx method.

INTRODUCTION AND DEVELOPMENT OF THE METHOD

Aquifers surround many oil and gas reservoirs. A number of methods have been developed for predicting water influx in to a reservoir. The present study is concerned with development of an improved method for prediction of radial/linear aquifer performance and group of parameters which control behaviour of a reservoir-aquifer system.

Ramey and Argarwal (1972) provide analysis for behaviour of a single well in an infinite homogeneous reservoir which includes consideration of compressibility of wellbore fluids. The same techniques can be used to describe behaviour of a reservoir surrounded by an infinite aquifer. The problem considered is radial flow of a slightly compressible fluid in an isotropic medium. The medium (aquifer) is considered infinite in extent with initial constant pressure P_i. At the inner boundary the aquifer is connected to a reservoir of radius r_{res}.

The diffusivity equation describing radial fluid flow can be expressed in terms of dimensionless variables as:

$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D} \times \frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \tag{1}$$

The intial and outer boundary conditions are respectively:

$$P_D\,(r_D,0)=0$$

$$\lim_{r_D \to \infty} P_D\,(r_D, t_D)=0 \tag{2}$$

The inner boundary condition is obtained from the equation describing transient pressure in the reservoir:

$$\frac{\partial P_{D_{res}}}{\partial t_D} - \left(\frac{\partial P_D}{\partial r_D}\right)_{r_D=1} = 1 \tag{3}$$

This boundary condition provides the equation relating constant surface flow rate and sand face flow rate:

$$\frac{q_{sf}}{q} - \left(\frac{\partial P_D}{\partial r_D}\right)_{r_D=1} = 1 - C_D \frac{\partial P_{D_{res}}}{\partial t_D} \tag{4}$$

Khairkhah (1975) shows values of P_D and $\partial P_D / \partial t_D$ which were calculated from values reported in Wattenbarger and Ramey (1970) and Ramey and Argarwal (1972). These values are used for the case of a reservoir surrounded by infinite radial aquifer.

The dimensionless variables in the previous equations are defined as follows:

Dimensionless time

$$t_D = \frac{0.000264\ K_\Omega\, t}{\phi_\Omega \mu_w (C_t)_\Omega r^2_{res}} \tag{5}$$

Dimensionless pressure

$$P_D = \frac{K_\Omega h_\Omega (P_i - P_{res})}{141.4\ q_{res}\ B_{res}\ \mu_w} \tag{6}$$

Dimensionless storage constant

$$C_D = \frac{(C_t)_{res} V_{res}}{2\pi h_\Omega \phi_\Omega (C_t)_\Omega r^2_{res}} \tag{7}$$

Dimensionless radius

$$r_D = \frac{r}{r_{res}} \tag{8}$$

Examination of the solutions showed that the behaviour of this radial - aquifer system is controlled by three groups of parameters:

(a) Reservoir compressibility
$A_1 = (C_t)_{res} r^2_{res} \phi_{res} h_\Omega$

(b) Aquifer compressibility (pseudo)
$A_2 = \phi_\Omega (C_t)^2_\Omega r_{res} h_\Omega$

(c) Reservoir - aquifer conductivity
$$A_3 = \frac{K_\Omega h_\Omega}{\mu_w}$$

Fig. 1 depicts a linear reservoir-aquifer system of the type considered here. The pressure in the system in terms of dimensionless variables is given by:

$$\frac{\partial^2 P_D}{\partial X^2} = \frac{\partial P_D}{\partial t_D} \quad \text{for } 0<X<1 \tag{9}$$

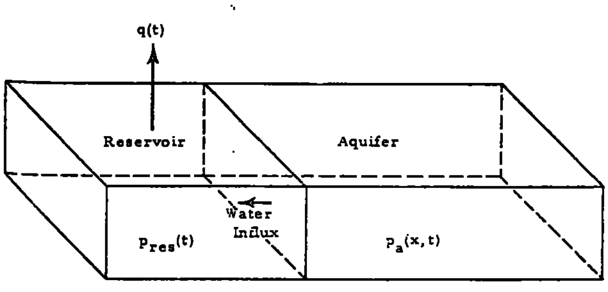

Fig. 1: Linear reservoir-aquifer system.

Initial and outer boundary conditions are respectively:

$$P_D(X, 0) = 0$$
$$P_D(1, t_D) = 0 \tag{10}$$

At the inner boundary the linear counterpart of the equation (3) is:

$$C_D \frac{\partial P_{D_{res}}}{\partial t_D} = \left(\frac{\partial P_D}{\partial X}\right)_{x=0} + 1 \tag{11}$$

where

$$t_D = \frac{0.000264\ K_\Omega\, t}{\phi_\Omega (C_t)_\Omega \mu_w L^2_\Omega} \tag{12}$$

$$P_D = \frac{0.001127\ K_\Omega A (P_i - P)}{q_{res} B_{res} \mu_w L_\Omega} \tag{13}$$

$$C_D = \frac{(C_t)_{res} V_{res}}{(C_t)_\Omega V_\Omega} \tag{14}$$

$$X = \frac{x}{L_\Omega} \tag{15}$$

Since no solution for linear aquifer comparable to that given in Ramey and Argarwal (1972) for radial aquifer was available in the literature, such a solution was derived. Khairkhah, (1975) shows the detailed derivation. Here we present the equation obtained giving reduced pressure in the reservoir as a function of reduced time for the case in which C_D and production rate are constant.

$$P_D(t_D, C_D) = 1 - \sum_{n=1}^{\infty} \frac{2e^{-z^2_n t_D}}{Z^2_n (1 + C_D + C^2_D Z^2_n)} \tag{16}$$

where Z_n's are roots of the equation

$$\tan Z_n = \frac{1}{C_D Z_n} \tag{17}$$

A computer program was written to evaluate $P_D(t_D, C_D)$ as given by equation (8). The results are presented in Fig. 2. Cumulative water influx in dimensionless form is given by:

$$Q_R = \frac{Q_w K_\Omega}{158.06\ q_{res} B_{res} \phi_{res} (C_t)_\Omega \mu_w L_\Omega^2} = \int_0^{t_D} \left(1 - C_D \frac{dP_D}{dt_D}\right) dt_D \tag{18}$$

An analysis similar to that done for radial systems discussed above shows that for a linear reservoir - aquifer system, the three groups of parameters which control performance of the system are:

(a) Reservoir compressibility:
$A_1 = (C_t)_{res} V_{res}$

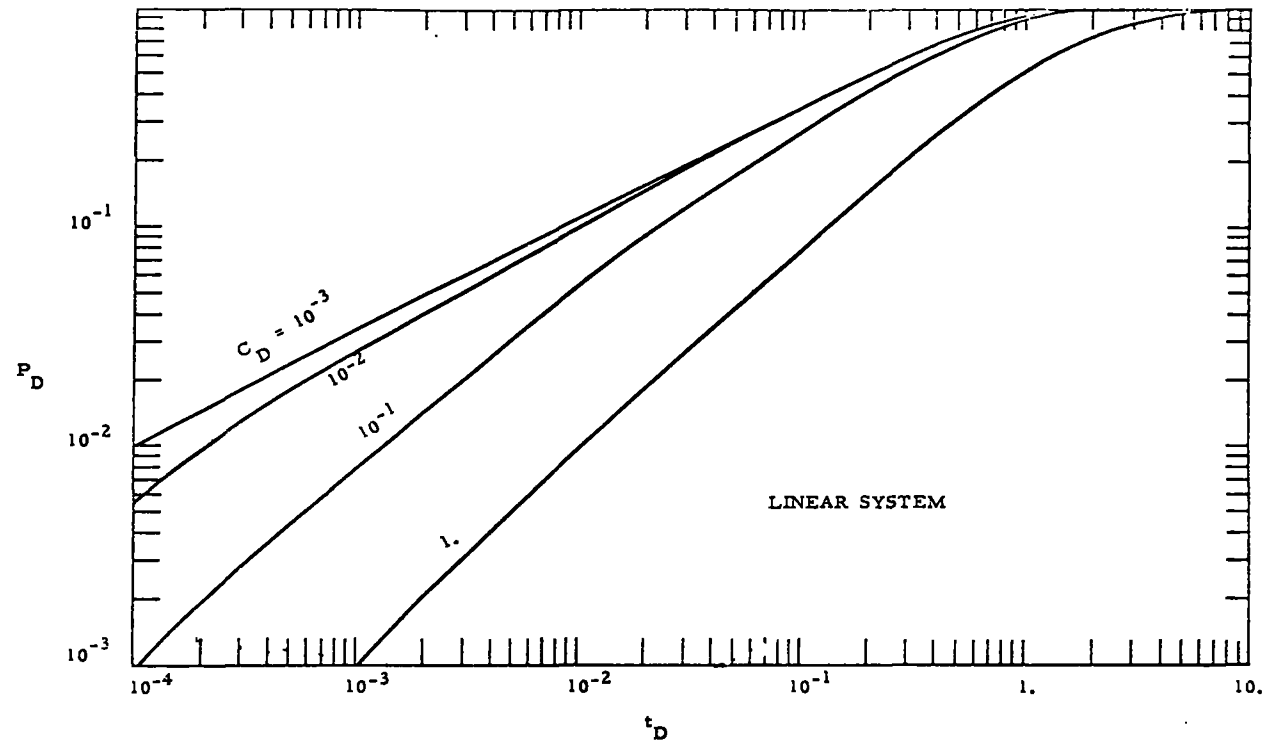

Fig. 2: $-P_D$ vs t_D for Linear reservoir-aquifer system.

(b) Aquifer compressibility:

$$A_2 = (C_t)_\Omega \ V_\Omega$$

(c) Reservoir - aquifer conductivity:

$$A_3 = \frac{K_\Omega A}{\mu_w L_\Omega}$$

The solutions presented above are for constant production rate and constant C_D. The case in which both C_D and production rate change continuously can be approximated by simultaneously considering n time increments within each of which flow rate q_i, is constant and m time increments within each of which reservoir compressibility, C_{Dj}, is constant.

$$P_{D_{res}}(t_D) = \sum_{i=1}^{n} (\hat{q}_{i+1} - \hat{q}_i) P_D (t_D - T_{Di}, C_{Dm}) + \sum_{j=1}^{m-1} P_{D_{res}} (t_{Dj}) (C_{D,j+1} - C_{Dj}) \frac{dP_D(t_D, C_{Dm})}{dt_D} \quad (19)$$

The respective dimensionless water influx is

$$Q_R = \int_o^{t_D} \hat{q} \, dt_D - \sum_{k=i}^{n-1} (C_{DK} - C_{D,K+1}) P_{D_{res}} (t_{Dk}) - C_{Dn} P_{D_{res}} (t_D) \quad (20)$$

A computer model was developed based on our present method and the validity of the approach and the magnitude of allowable changes in C_D were determined by comparing results to those obtained with more exact tank - type simulator (Dougherty and Mitchell, 1963).

The accuracy of the tank type simulator was established by comparing calculated results to values from the analytical solution for both linear and radial aquifer.

A series of tests were run to establish the limits of changes in C_{Dj} which still can produce satisfactory results and this constraint was observed in the subsequent calculations.

After the accuracy of the method had been established, it was incorporated into a computer model for estimating reservoir parameters. This model determines *optimal* values of reservoir parameters by modifying an efficient optimization method (Hernandez and Swift, 1972). The main thrust of our modification is establishing a weighting system for each set of random values generated.

APPLICATION OF THE METHOD

The method developed in the present study has been applied to different oil and gas reservoir problems. For brevity, only one of these applications is presented here. The reservoir of interest is undersaturated Middle Marge Tex Oil Zone in Parish, La. Figs 3 and 4 show the structure of the reservoir. The result of the application of the Hurst - Van Everdingen radial unsteady - state method to this field is shown in Fig. 5. For comparison, we applied our method to the same problem for both linear and radial aquifers. We used part of the available production history for estimation of the reservoir performance during the unused production history period. The results are shown in Figs. 6 and 7. Below we present the optimized reservoir - aquifer values obtained from our model:

	Radial	Linear
A1	70 x 10^6	70 x 10^6
A2	146	95 000
A3	1228	1200

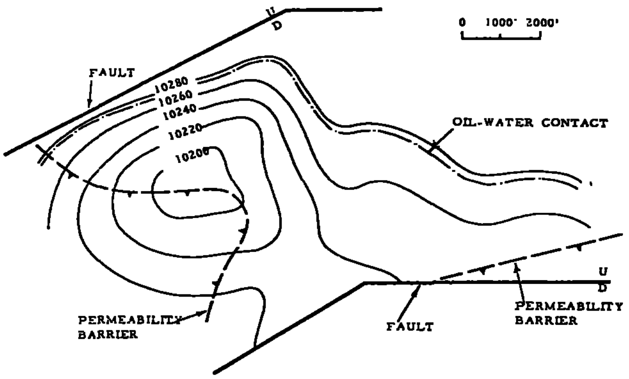

Fig. 3: Structure of Middle Marge Tex reservoir.

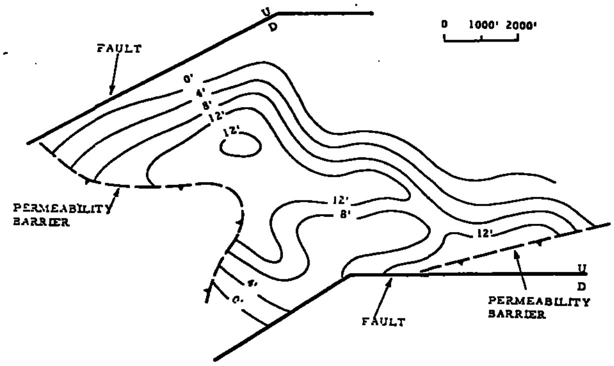

Fig. 4: Middle Marge Tex reservoir.

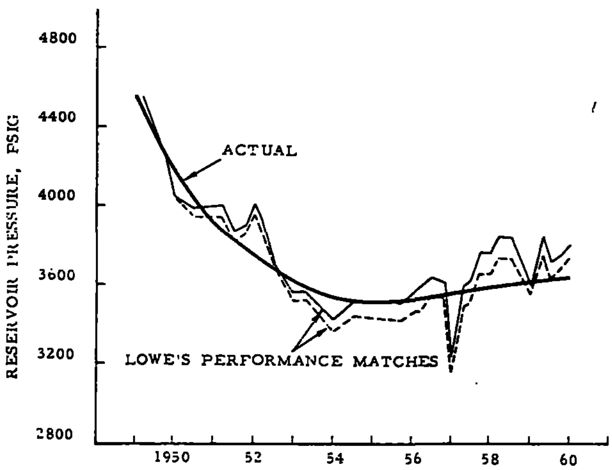

Fig. 5: Performance matches reported in Lowe (1967).

A comparison of the results indicates that a radial aquifer will give much better agreement for all parameter values considered. It is also shown that reservoir volume of 70 x 10^6 ft^3 which is obtained from initial tank - oil - in place estimate, produces smallest pressure error for radial aquifer.

CONCLUSIONS

(a) A method for study of transient behaviour of oil/gas reservoir with linear or (infinite) radial aquifers is presented.

(b) The method and our computer models can be applied to reservoirs to predict future performance and estimate optimized three controlling reservoir - aquifer parameters.

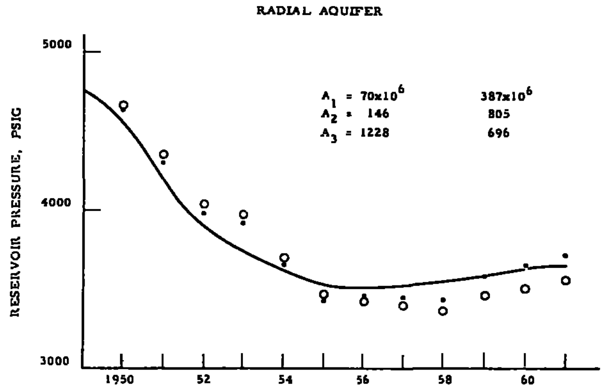

Fig. 6: Performance matches obtained with radial aquifer.

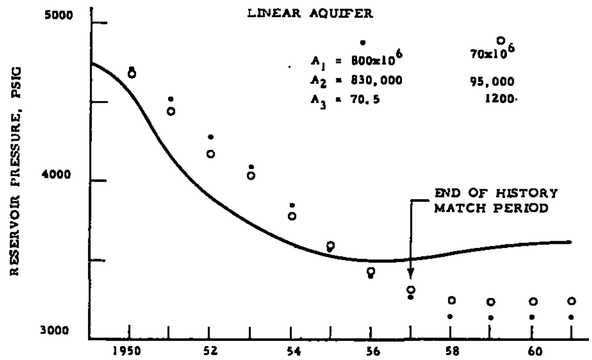

Fig. 7: Performance matches obtained with linear aquifer.

(c) Both linear and radial aquifers can produce comparable history matches.

NOMENCLATURE

A = Flow are between reservoir and aquifer, ft^3
A_1 = Controlling parameter of reservoir-aquifer system, ft^3
A_2 =Controlling parameter of reservoir-aquifer system, ft^2/psi
A_3 = Controlling parameter of reservoir-aquifer system, (md - ft)/cp
B = Reservoir fluid formation volume factor, BBL. at reservoir condition/BBL. at standard condition.
C_D = Dimensionless storage constant [Eq. (A34)]
C = Compressibility, psi^{-1}
C_t = Total compressibility of system, psi^{-1}
e = Base of natural logarithm
h = Thickness, ft
K = Permeability, md
L = Length of linear system, ft
P = Pressure, psi
P_i = Initial pressure, psi
P_D = Dimensionless pressure
q = Flow rate at surface condition, B/D
q_{sf} = Sand face flow rate, B/D
$\hat{q}$ = $(q_{res} . B_{res}) / (q^o_{res} . B^o_{res})$, dimensionless flow rate
Q_w = Cumulative water encroachment, BBL

Q_R = Dimensionless cumulative water encroachment
r = Radial distance, ft
r_D = Dimensionless radius
t = Time, hr
t_D = Dimensionless time
tan = Tangent function
V = Volume, ft^3
X = Dimensionless linear distance
x = Linear distance of point, located in aquifer, from reservoir aquifer boundary, ft
Z_n = Roots of tan $Z_n = 1/(C_D Z_n)$
μ_w = Water viscosity, cp
T_D = Dimensionless time at which rate changes
¶ = Porosity

Subscripts
a = Aquifer
D = Dimensionless
res = Reservoir

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